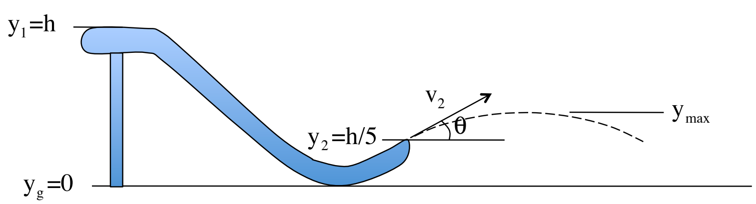
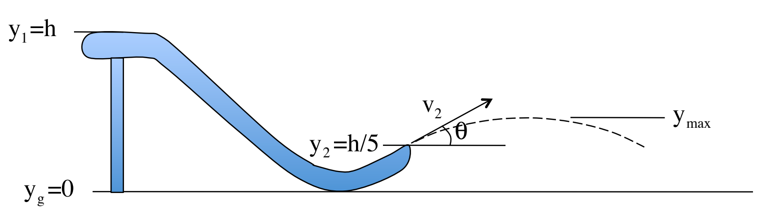
Problem 8.27

For easy determination of "y" parameters, I've made the sketch large.

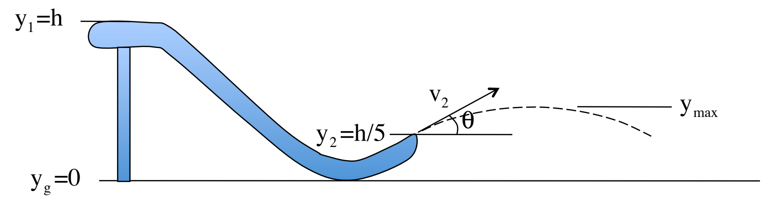


The problem asks for the kid's maximum height y_{max} after leaving the slide and "free falling." Bare bone, this is a *Modified Conservation of Energy* problem with a slight twist. The velocity at y_{max} is equal to the *x-component* of the velocity when the body became free, which is to say at y_2. In other words, if you ran into this on an AP test, you'd use *Conservation of Energy* to determine the velocity when the kid left the slide (v_2), determine that velocities x-component, then use *Conservation of Energy* again to determine y_{max}. *This* problem, on the other hand, is having you do everything in steps. Sooooo . . .



a.) Yes, the child/earth system is "isolated."

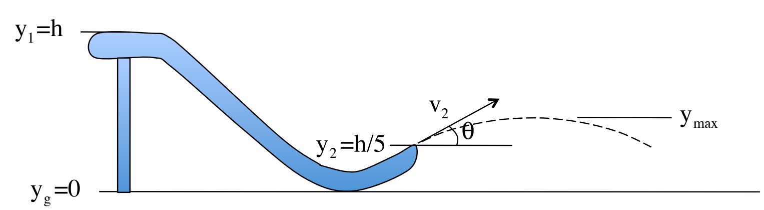
Additional comment: A system is *isolated*, by definition, if no mechanical energy is "lost" during the motion (that is, if no mechanical energy leaks away by "crossing the boundary of the system"). Frictional effects turn kinetic energy into heat, so if friction existed (which it doesn't in this case), the total mechanical energy of the system would not remain constant throughout time and the system would not be isolated. In this case, there is no friction to pull energy out of the system, and the only work being done is due to gravity (the normal does none as it is perpendicular to the motion, and we are ignoring air friction). Only gravity does work, and earth is a part of the system.



b.) There are no non-conservative forces in this system as friction, the most common of that ilk, is being ignored here.

c.) The total mechanical energy in the system is the sum of kinetic and potential energy *at any point during the body's motion*. If mechanical energy is conserved (this is the case when we have *potential energy functions* for all work-producing forces in the system, and no *non-conservative forces* like friction), the total mechanical energy never changes. This is the case in this problem. As we know the kid starts from rest, there is initially no kinetic energy. That means all the initial mechanical energy is wrapped up in gravitational potential energy. In other words:

$$E_1 = mgh$$



d.) The total mechanical energy in the system doesn't change (no friction), so one way expressing the total mechanical energy at the launch point is still:

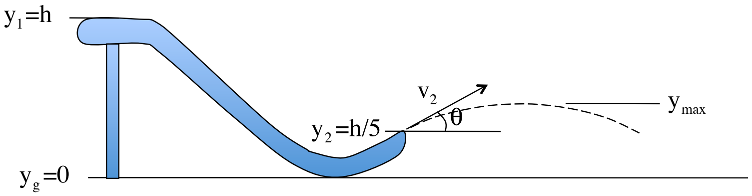
$$E_2 = E_1 = mgh$$

We could, though, also write it as the sum of the kinetic and potential energies at that point. That is:

$$E_2 = \frac{1}{2}mv_2{}^2 + mg\left(\frac{h}{5}\right)$$

e.) Not to be a broken record, but the total mechanical energy hasn't changed, so:

$$E_{top} = \frac{1}{2}mv_{top}{}^2 + mg\left(y_{max}\right) = E_2 = \frac{1}{2}mv_2{}^2 + mg\left(\frac{h}{5}\right) = E_1 = mgh$$



f.) The velocity at the launch point:

Picking out the appropriate relationship (in red) from the expression listed at the bottom of the preceding page, we get:

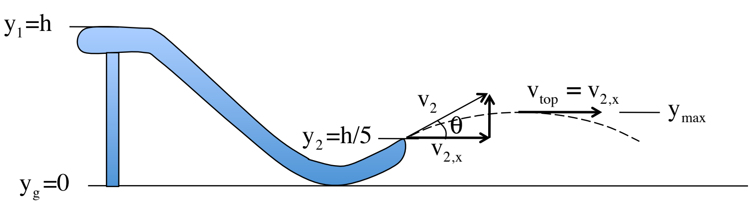
$$E_{top} = \frac{1}{2}mv_{top}^{\ 2} + mg(y_{max}) = E_2 = \frac{1}{2}mv_2^{\ 2} + mg\left(\frac{h}{5}\right) = E_1 = mgh$$

That is:

$$\frac{1}{2}\not{m}v_2^{\ 2} + \not{m}g\left(\frac{h}{5}\right) = \not{m}gh$$

$$\Rightarrow \quad v_2 = \left[2\left(gh - g\left(\frac{h}{5}\right)\right)\right]^{1/2}$$

$$= \left(\frac{8}{5}gh\right)^{1/2}$$



g.) What is the maximum height after launch?

We are going to use *conservation of energy* again, but there is a trick to the proceedings. We need the magnitude of the velocity at the top of the free fall arc, which we know from experience is the magnitude of the x-component of the motion at that point. That, though, is the same for *any* point during the fall. We know the velocity magnitude and angle at the launch point, $v_2 = \sqrt{\tfrac{8}{5}gh}$, so we can use that to determine that component and write:

X-component at launch is: $v_{2,x} = v_2\cos\theta = \left(\tfrac{8}{5}gh\right)^{1/2}\cos\theta$

so that x-component at top is: $v_{top} = v_{2,x} = \left(\tfrac{8}{5}gh\right)^{1/2}\cos\theta$

With that, and again picking the right relationship combination (in red):

$$E_{top} = \frac{1}{2}mv_{top}^{\ 2} + mg(y_{max}) = E_2 = \frac{1}{2}mv_2^{\ 2} + mg\left(\frac{h}{5}\right) = E_1 = mgh$$

$$\Rightarrow \quad y_{max} = \left[\frac{\not{m}gh - \frac{1}{2}\not{m}v_{top}^{\ 2}}{\not{m}g}\right]$$

$$= \left[\frac{gh - \frac{1}{2}\left[\left(\tfrac{8}{5}gh\right)^{1/2}\cos\theta\right]^2}{g}\right]$$

$$= \left[\frac{gh - \frac{1}{2}\left(\tfrac{8}{5}gh\right)\cos^2\theta}{g}\right]$$

$$= h\left(1 - \frac{4}{5}\cos^2\theta\right)$$